

On the electromagnetic hadron current derived from the gauged Wess-Zumino-Witten action

M. Wakamatsu*

*Department of Physics, Faculty of Science,
Osaka University,
Toyonaka, Osaka 560-0043, JAPAN*

Abstract

In a recent paper, based on the Skyrme model with the Wess-Zumino-Witten term including electromagnetism, Eto et al. pointed out an intriguing possibility that the Gell-Mann-Nishijima relation is corrected under the presence of back-ground electromagnetic fields, thereby being led to the conclusion that even a neutron acquires non-zero net charge in external magnetic fields. We point out that this remarkable conclusion is inseparably connected with an unwelcome feature of the gauged Wess-Zumino-Witten action, i.e. the non-conservation of source current of Maxwell equation.

* wakamatu@phys.sci.osaka-u.ac.jp

I. INTRODUCTION

In a recent paper [1], Eto et al. investigated electromagnetic properties of baryons under the influence of external electromagnetic field, based on the Skyrme model [2] with Wess-Zumino-Witten term including electromagnetism [3],[4], thereby concluding that a nucleon in external electromagnetic fields has anomalous charge distribution due to the chiral anomaly. Furthermore, the Gell-Mann-Nishijima relation, $Q = I_3 + N_B/2$ (Q : electric charge, I_3 : the third component of isospin, N_B : baryon number), acquires an additional term due to the quantum anomaly. As a consequence, non-zero net charge, which is generally non-integer, is induced even for a neutron. This astounding conclusion stems from the gauged Wess-Zumino-Witten action with two flavors, given in the form [5],[6] :

$$S_{WZW}[U, A_\mu] = -e \int d^4x A_\mu \left(\frac{N_c}{6} j_B^\mu + \frac{1}{2} j_{anm}^\mu \right), \quad (1)$$

where

$$j_B^\mu = -\frac{1}{24\pi^2} \epsilon^{\mu\nu\alpha\beta} \text{tr} (L_\nu L_\alpha L_\beta), \quad (2)$$

$$j_{anm}^\mu = \frac{ieN_c}{96\pi^2} \epsilon^{\mu\nu\alpha\beta} F_{\nu\alpha} \text{tr} \tau_3 (L_\beta + R_\beta), \quad (3)$$

with

$$L_\mu \equiv U \partial_\mu U^\dagger, \quad R_\mu \equiv \partial_\mu U^\dagger U. \quad (4)$$

(We point out that our definition of L_μ and R_μ is different from that in [1].) Here, j_B^μ is the well-known baryon current giving an integer baryon number [4]. According to [1], in the presence of background electromagnetic fields, not only the first term but also the second term of Eq. (1) is important. The electric charge Q with the contribution of anomaly is then written as

$$Q = I_3 + \frac{N_B}{2} + \frac{Q_{anm}}{2}, \quad (5)$$

where $N_B = \int d^3x j_B^0$ and $Q_{anm} = \int d^3x j_{anm}^0$. This means that the Gell-Mann-Nishijima relation receives a remarkable modification under the background electromagnetic fields.

It appears to us, however, that the above-mentioned anomalous induction of non-zero net charge for a nucleon (or a Skyrmion) is not in good harmony with the schematic physical picture illustrated in Fig.1 of their paper. This schematic diagram represents electric charge generation of a nucleon through the anomalous coupling between one pion and two

photons (or electromagnetic fields). Since the electromagnetic fields (as abelian gauge fields) carries no electric charge, the exchanged pion in this figure must be neutral. In fact, this lowest order diagram results from the same vertex as describing the famous decay process $\pi^0 \rightarrow 2\gamma$ due to the triangle anomaly [7],[8], which is legitimately contained in the gauged Wess-Zumino-Witten action [4]-[6]. Naturally, the gauged Wess-Zumino-Witten action also contains higher-power terms in the pion fields. However, even if one considers diagrams in which more pions are exchanged between the nucleon and the electromagnetic fields, the exchanged pions must be electrically neutral as a whole, since the electromagnetic fields carry no electric charge. What we are worrying about here is a conflict between this intuitive thought and the principle conclusion of the paper [1], i.e. the anomalous induction of non-zero net charge for a nucleon.

The purpose of the present paper is to unravel the origin of this contradiction. Here, we unavoidably encounter the problem of how to define electromagnetic hadron current in an unambiguous manner by starting with the gauged Wess-Zumino action. A subtlety arises from the fact that the gauged Wess-Zumino Witten action contains nonlinear terms in the electromagnetic fields. In fact, if it contains only linear terms in the electromagnetic fields, it is clear that one can easily read off the electromagnetic hadron current as a coefficient of the electromagnetic field. For handling this delicate point, first in sect.II, we briefly analyze the familiar lagrangian of scalar electrodynamics containing couplings between photons and complex scalar fields, which is nonlinear in the photon fields. A particular emphasis here is put on how to define electromagnetic matter current based on a solid guiding principle. It will be shown there that the two forms of current, i.e. the one defined on the basis of the Noether theorem and the other defined as a source current of the Maxwell equation through the equations of motion, perfectly coincides with each other. It is also shown that this current is gauge-invariant and conserved, thereby ensuring the consistency of scalar electrodynamics as a quantum gauge theory. In section III, we shall carry out a similar analysis for the nonlinear meson action with the Wess-Zumino-Witten action including electromagnetism to find something unexpected, which is thought to be the origin of the discrepancy pointed out above. Finally, in sect. IV, we briefly summarize what we have found in the present paper.

II. A LESSON LEARNED FROM SCALAR ELECTRODYNAMICS

Let us start with the familiar lagrangian of scalar electrodynamics given by

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + D_\mu \phi^* D^\mu \phi - V(\phi^* \phi), \quad (6)$$

with

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu, \quad (7)$$

$$D^\mu \phi(x) \equiv [\partial^\mu + i e A^\mu(x)] \phi(x). \quad (8)$$

This lagrangian is manifestly gauge-invariant under the following gauge transformation :

$$\phi(x) \rightarrow e^{-i e \alpha(x)} \phi(x), \quad A^\mu(x) \rightarrow A^\mu(x) + \partial^\mu \alpha(x). \quad (9)$$

The equations of motion derived from the above lagrangian are given by

$$\partial_\mu F^{\mu\nu} = j^\nu, \quad (10)$$

$$D_\mu D^\mu \phi = -\partial V / \partial \phi^*, \quad (11)$$

$$(D_\mu D^\mu \phi)^* = -\partial V / \partial \phi. \quad (12)$$

Here, the source current j^ν of the Maxwell equation (10) is given by

$$j^\nu = i e [\phi^* D^\mu \phi - (D^\mu \phi)^* \phi] = i e \phi^* \overleftrightarrow{\partial}^\nu \phi - 2 e^2 \phi^* \phi A^\nu, \quad (13)$$

with $\overleftrightarrow{\partial}^\nu = \overrightarrow{\partial}^\nu - \overleftarrow{\partial}^\nu$. By using the equations of motion, it can be shown that this matter current j^ν is conserved, i.e.

$$\partial_\nu j^\nu = 0. \quad (14)$$

One can also convince that this matter (or source) current is invariant under the full gauge transformation (9). One should recognize that the conservation of source current is crucially important. In fact, if it were broken, one encounters a serious contradiction with the Maxwell equation (10) in such a way that

$$0 = \partial_\nu \partial_\mu F^{\mu\nu} = \partial_\nu j^\nu \neq 0. \quad (15)$$

For later discussion, it is useful to remember the fact that the matter current above can also be obtained by using the standard Noether prescription. To confirm it, we first consider the infinitesimal version of the gauge transformation given by

$$\delta \phi = -i e \epsilon(x) \phi, \quad \delta \phi^* = i e \epsilon(x) \phi^*, \quad \delta A^\mu = \partial^\mu \epsilon(x). \quad (16)$$

Naturally, the lagrangian of the scalar electrodynamics is invariant under this gauge transformation. The Noether current is obtained by considering another variation

$$\delta' \phi = -i e \epsilon(x) \phi, \quad \delta' \phi^* = i e \epsilon(x) \phi^*, \quad \delta' A^\mu = 0. \quad (17)$$

The variation of the whole lagrangian under this transformation is reduced to the form

$$\delta' \mathcal{L} = \epsilon(x) \partial_\mu J^\mu + \partial_\mu \epsilon(x) J^\mu, \quad (18)$$

which defines the current J^μ such a way that

$$J^\mu = \frac{\partial(\delta' \mathcal{L})}{\partial(\partial_\mu \epsilon(x))}, \quad (19)$$

$$\partial_\mu J^\mu = \frac{\partial(\delta' \mathcal{L})}{\partial \epsilon(x)}. \quad (20)$$

If the lagrangian \mathcal{L} is invariant under a space-time independent (global) transformation

$$\delta' \phi = -i e \epsilon \phi, \quad \delta' \phi^* = i e \epsilon \phi^*, \quad (21)$$

which is indeed the case with our lagrangian (1), we conclude that

$$0 = \delta' \mathcal{L} = \epsilon \partial_\mu J^\mu. \quad (22)$$

This means that the current J^μ defined by the equation (19) is in fact a conserved Noether current. The above-explained method of obtaining the Noether current is known as the Gell-Mann-Levy method.

Now, for the lagrangian of the scalar electrodynamics, we have

$$\delta' [\partial_\mu \phi^* \partial^\mu \phi] = i e \partial_\mu \epsilon(x) [\phi^* \partial^\mu \phi - \partial^\mu \phi^* \phi], \quad (23)$$

$$\delta' [i e (\partial_\mu \phi^* \phi - \phi^* \partial_\mu \phi) A^\mu] = \partial_\mu \epsilon(x) (-2 e^2) \phi^* \phi A^\mu, \quad (24)$$

$$\delta' [e^2 A_\mu A^\mu \phi^* \phi] = 0, \quad (25)$$

thereby being led to

$$\delta' \mathcal{L} = \partial_\mu \epsilon(x) \{ i e [\phi^* \partial^\mu \phi - (\partial^\mu \phi)^* \phi] - 2 e^2 \phi^* \phi A^\mu \}. \quad (26)$$

The resultant Noether current is therefore given by

$$J^\mu = i e \phi^* \overleftrightarrow{\partial}^\mu \phi - 2 e^2 \phi^* \phi A^\mu = i e [\phi^* D^\mu \phi - (D^\mu \phi)^* \phi]. \quad (27)$$

One confirms that this conserved Noether current precisely coincides with the source current appearing in the Maxwell equation (10) for the photon field. This ensures the consistency of the scalar electrodynamics as a classical and a quantum field theory. Somewhat embarrassing, we shall see below that the familiar gauged Wess-Zumino-Witten action does not satisfy the same sense of consistency.

III. ELECTROMAGNETIC HADRON CURRENT RESULTING FROM THE GAUGED WESS-ZUMINO-WITTEN ACTION

A. Matter current derived from Noether principle

Here, we start with the gauged Wess-Zumino-Witten action with two flavor expressed in the following form :

$$\begin{aligned}
S_{WZW}[U, A_\mu] = & S_{WZ}[U] - \frac{1}{2} e \int d^4x A_\mu \\
& \times \left\{ -\frac{1}{24 \pi^2} \epsilon^{\mu\nu\alpha\beta} \text{tr} (L_\nu L_\alpha L_\beta) \right. \\
& \left. + \frac{3 i e}{48 \pi^2} \epsilon^{\mu\nu\alpha\beta} F_{\nu\alpha} \text{tr} Q (L_\beta + R_\beta) \right\}, \tag{28}
\end{aligned}$$

where Q is the $SU(2)$ charge matrix given as

$$Q = \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix}. \tag{29}$$

(As is well-known, although $S_{WZ}[U]$ vanishes in the $SU(2)$ case, its gauge variation does not. We therefore retain it here.) By construction, i.e. as a consequence of the “trial and error” gauging *a la* Witten [4], the gauged Wess-Zumino-Witten action is invariant under the following infinitesimal gauge transformation :

$$\delta U = i \epsilon(x) [Q, U], \quad \delta U^\dagger = i \epsilon(x) [Q, U^\dagger], \quad \delta A_\mu = -\frac{1}{e} \partial_\mu \epsilon(x). \tag{30}$$

Let us first try to see what answer we shall obtain for the Noether current, if we apply the Gell-Mann-Levy method to the above lagrangian (28). The transformation, which we consider to this end, is given by

$$\delta' U = i \epsilon(x) [Q, U], \quad \delta' U^\dagger = i \epsilon(x) [Q, U^\dagger], \quad \delta' A_\mu = 0. \tag{31}$$

Making use of the relation

$$\begin{aligned}
\delta' S_{WZW} = & - \int d^4x \partial_\mu \epsilon(x) \frac{1}{48 \pi^2} \epsilon^{\mu\nu\alpha\beta} \text{tr} (L_\nu L_\alpha L_\beta) \\
& - \int d^4x \partial_\nu \epsilon(x) \frac{3 i e}{48 \pi^2} \epsilon^{\mu\nu\alpha\beta} A_\mu \partial_\alpha \text{tr} Q (L_\beta + R_\beta), \tag{32}
\end{aligned}$$

we readily find that the corresponding Noether current is given by

$$J_1^\mu \equiv \frac{\delta(\delta' S_{WZW})}{\delta(\partial_\mu \epsilon(x))} = -\frac{1}{48\pi^2} \epsilon^{\mu\nu\alpha\beta} \text{tr}(L_\nu L_\alpha L_\beta) + \frac{3ie}{48\pi^2} \epsilon^{\mu\nu\alpha\beta} A_\nu \partial_\alpha \text{tr} Q(L_\beta + R_\beta). \quad (33)$$

One can also verify that this current is invariant under the full gauge transformation (30). Unfortunately, this current is not conserved. In fact, we find that

$$\partial_\mu J_1^\mu = \frac{3ie}{24\pi^2} \epsilon^{\mu\nu\alpha\beta} \partial_\mu A_\nu \partial_\alpha \text{tr} Q(L_\beta + R_\beta) \neq 0. \quad (34)$$

However, one can verify that the r.h.s. of (34) is a total derivative of another four-vector as

$$\partial_\mu J_1^\mu = \partial_\mu X^\mu, \quad (35)$$

with

$$X^\mu \equiv \frac{3ie}{48\pi^2} \epsilon^{\mu\nu\alpha\beta} A_\nu \partial_\alpha \text{tr} Q(L_\beta + R_\beta). \quad (36)$$

This means that, if we define another current j_{II}^μ by

$$J_{\text{II}}^\mu \equiv J_1^\mu - X^\mu = -\frac{1}{24\pi^2} \epsilon^{\mu\nu\alpha\beta} \text{tr}(L_\nu L_\alpha L_\beta), \quad (37)$$

then, J_{II}^μ is conserved. The price to pay is that the new current J_{II}^μ is no longer gauge-invariant.

Incidentally, in the case of Poincare symmetry not of internal symmetry, the ambiguous nature of the Noether current is widely known. For example, in the case of quantum chromodynamics (QCD), the 2nd-rank energy momentum tensor obtained from a naive Noether procedure does not satisfy the desired symmetry property under the exchange of two Lorentz indices [10]. However, there exists a well-known procedure for “improving” the Noether current by adding a superpotential - divergence of anti-symmetric tensor - which does not spoil the current conservation. The symmetric energy momentum tensor of QCD obtained in such a procedure is sometimes called Belinfante symmetrized energy-momentum tensor.

Summarizing the analysis in this subsection, we have applied the familiar Gell-Mann-Levy method to the gauged Wess-Zumino-Witten action for obtaining a Noether current as a candidate of electromagnetic hadron current. However, we have ended up with two different forms of currents, i.e. J_1^μ and J_{II}^μ . The current J_1^μ is gauge-invariant but not conserved, while the current J_{II}^μ is conserved but not gauge-invariant. As pointed out in the

paper by Son and Stephanov [9], one can construct the 3rd current, which satisfies both of gauge-invariance and current conservation, by using the “trial and error” gauging method as proposed by Witten. It is given by

$$J_{\text{III}}^\mu = -\frac{1}{48\pi^2} \epsilon^{\mu\nu\alpha\beta} \text{tr}(L_\nu L_\alpha L_\beta) - \frac{3ie}{48\pi^2} \epsilon^{\mu\nu\alpha\beta} \partial_\nu [A_\alpha \text{tr} Q (L_\beta + R_\beta)]. \quad (38)$$

Unfortunately, it is not a current derived from the gauged Wess-Zumino-Witten action on the basis of a definite prescription as guided by the Noether principle.

In this way, we must conclude that, quite different from the case of scalar electrodynamics, the standard Noether method does not do a desired good job to derive the electromagnetic hadron current corresponding to the gauged Wess-Zumino-Witten action, in the sense that it fails to give a candidate of electromagnetic hadron current, satisfying both of gauge-invariance and conservation. In the next subsection, we shall investigate the nature of another candidate of electromagnetic hadron current, i.e. the source current, which is defined through the equation of motion for the electromagnetic field.

B. Matter current as a source of Maxwell equation

The full action of the two-flavor Skyrme model coupled to the electromagnetic field A_μ is given by

$$S = S_\gamma[A_\mu] + S_{\text{Skyrme}}[U, A_\mu] + S_{\text{WZW}}[U, A_\mu]. \quad (39)$$

Here, the 1st term

$$S_\gamma = -\frac{1}{4} \int d^4x F_{\mu\nu} F^{\mu\nu} \quad (40)$$

is the kinetic part of the electromagnetic field, while the 2nd term, $S_{\text{Skyrme}}[U, A_\mu]$, stands for the non-anomalous part of action for the two-flavor Skyrme model minimally coupled to the electromagnetic field. The 3rd term, i.e. $S_{\text{WZW}}[U, A_\mu]$, gives the gauged Wess-Zumino-Witten action given by (28). In the following, we shall discard the part $S_{\text{Skyrme}}[U, A_\mu]$ for simplicity, since it plays no essential role in our discussion below. The Euler-Lagrange equation of motion for the electromagnetic field is therefore written down from

$$\frac{\delta}{\delta A_\nu} \{ S_\gamma[A_\mu] + S_{\text{WZW}}[U, A_\mu] \} = 0, \quad (41)$$

which gives the Maxwell equation

$$\partial_\mu F^{\mu\nu} = j^\nu, \quad (42)$$

with the definition of the source current j^ν as

$$e j^\nu \equiv - \frac{\delta}{\delta A_\nu} S_{WZW}[U, A_\mu]. \quad (43)$$

(Naturally, if we had included the part $S_{Skyrme}[U, A_\mu]$, it would also contribute to the source current of Maxwell equation. However, this part of current is conserved itself and does not cause any trouble as discussed below.) An immediate question is whether the above definition, given as a functional derivative of the gauged Wess-Zumino-Witten action with respect to the electromagnetic fields, offers us the same answer as obtained with the Noether prescription. The answer is no. We find that the source current is given by

$$\begin{aligned} j^\mu = & - \frac{1}{48 \pi^2} \epsilon^{\mu\nu\alpha\beta} \text{tr} (L_\nu L_\alpha L_\beta) \\ & + \frac{3 i e}{96 \pi^2} \epsilon^{\mu\nu\alpha\beta} F_{\nu\alpha} \text{tr} Q (L_\beta + R_\beta) \\ & + \frac{3 i e}{48 \pi^2} \epsilon^{\mu\nu\alpha\beta} \partial_\nu [A_\alpha \text{tr} Q (L_\beta + R_\beta)]. \end{aligned} \quad (44)$$

which does not coincide with any of the currents J_I^μ , J_{II}^μ , and J_{III}^μ discussed in the previous subsection. Somewhat unexpectedly, it turns out that this current j^μ is not gauge-invariant. More serious problem is that it is not conserved, owing to the presence of the 2nd term of (44). In fact, we find that

$$\partial_\mu j^\mu = \partial_\mu X^\mu \neq 0, \quad (45)$$

with

$$X^\mu = \frac{3 i e}{48 \pi^2} \epsilon^{\mu\nu\alpha\beta} A_\nu \partial_\alpha \text{tr} Q (L_\beta + R_\beta). \quad (46)$$

As emphasized in the example of scalar electrodynamics, non-conservation of source current is not permissible, since it causes an incompatibility with the fundamental equation of electromagnetism, i.e. the Maxwell equation [11]. How can we make a compromise with this trouble. One possible attitude would be to follow the argument as given by Kaymakçalan, Rajeev and Schechter many years ago [5]. They argue that the low-energy effective action for QCD involves many more new fields and interactions so one should not worry too much about the complete consistency of equation of motion. The effective action is, after all, being used as a handy mnemonic to read off the relevant vertices. The gauged Wess-Zumino-Witten action certainly describes the typical anomalous processes containing the photons

like $\pi^0 \rightarrow 2\gamma$ and/or $\gamma \rightarrow 3\pi$ consistently with the low energy theorem, i.e. the anomalous Ward identities.

Now we are in a position to pinpoint the origin of somewhat astounding conclusion obtained in the paper [1], i.e. the anomalous induction of net electric charge for a nucleon in the magnetic fields. This conclusion follows from the electromagnetic hadron current given as a half of the sum of j_B^μ in (2) and j_{ann}^μ in (3). Setting $N_c = 3$, this reduces to

$$j^\mu = -\frac{1}{48\pi^2} \epsilon^{\mu\nu\alpha\beta} \text{tr}(L_\nu L_\alpha L_\beta) + \frac{ieN_c}{192\pi^2} \epsilon^{\mu\nu\alpha\beta} F_{\nu\alpha} \text{tr} \tau_3 (L_\beta + R_\beta). \quad (47)$$

In consideration of the fact that $Q = \frac{1}{6} + \frac{\tau_3}{2}$, this current just coincides with the sum of the 1st and 2nd terms in the source current (47), which we have derived above. Since the 3rd term of the current (44) is of a total derivative form, it does not contribute to the net charge of a nucleon. We thus find that the 2nd term of the current (44) or of the current (47) is the cause of trouble, which prevents the conservation of source current of the Maxwell equation. In any case, what we can say definitely from the analysis above is that the anomalous induction of non-zero net charge for a nucleon (or a Skyrmion) claimed in the paper [1] is inseparably connected with this unfavorable feature of the gauged Wess-Zumino-Witten action. Still, what is lacking in our understanding is a deep explanation of why the gauged Wess-Zumino-Witten action, which was constructed so as to fulfill the electromagnetic gauge-invariance with use of the “trial and error” method, does not satisfy the consistency with the Maxwell equation.

A final comment is on a related work by Kharzeev, Yee, and Zahed [12], which was done motivated by the paper [1]. Starting with a simple effective lagrangian of QCD (it corresponds to the lowest power term in the pion field in the gauged Wess-Zumino-Witten action), they investigated the effect of anomaly induced charge distribution in the nucleon. Under a certain kinematical approximation concerning the classical equation of motion for the pion field in a nucleon, they conclude that the abelian anomaly of QCD induces a quadrupole moment for a neutron but it does not induce net electric charge for it. The last statement, i.e. no induction of net electric charge for a neutron appears to be consistent with the nature of their effective lagrangian and also with the intuitive consideration given in the introduction of the present paper.

IV. SUMMARY AND CONCLUSION

To conclude, motivated by the recent claim that, under the external magnetic fields, the anomalous couplings between mesons and electromagnetic fields contained in the gauged Wess-Zumino-Witten action induces non-zero net electric-charge for a nucleon, we have carefully re-investigated the problem of how to define the electromagnetic hadron current from this widely-known action. To this end, we first compare the two methods of obtaining electromagnetic hadron current for the familiar lagrangian of the scalar electrodynamics. The one is the Gell-Mann-Levy method to obtain the Noether current, while the other is the method of using equations of motion of actions to define source current. For this standardly-known lagrangian, we confirm that these two methods give precisely the same form of the electromagnetic hadron current. It can also be verified that this current is gauge-invariant and conserved. Unfortunately, this is not the case with the gauged Wess-Zumino-Witten action. That is, the currents obtained by these two methods do not coincide with each other. Particularly troublesome here is the fact that the source current of Maxwell equation is not conserved. This means that the gauged Wess-Zumino-Witten action, which was constructed so as to fulfill the electromagnetic gauge-invariance by using the “trial and error” method, does not satisfy the consistency with the the fundamental equation of electromagnetism. Although mysterious, it seems at the least clear that the recently claimed anomalous induction of net electric charge for a nucleon in the magnetic fields is inseparably connected with this unwelcome feature of the gauged Wess-Zumino-Witten action.

ACKNOWLEDGMENTS

The author would like to thank Prof. T. Kubota for useful discussion. This work is supported in part by a Grant-in-Aid for Scientific Research for Ministry of Education, Culture, Sports, Science and Technology, Japan (No. C-21540268)

-
- [1] M. Eto, K. Hashimoto, H. Iida, T. Ishii, and Y. Maezawa, arXiv : 1103.5443 [hep-ph].
 - [2] T.H.R. Skyrme, Proc. Roy. Soc. Lond. A **260**, 127 (1961).
 - [3] J. Wess and B. Zumino,, Phys. Lett. B **37**, 95 (1971).

- [4] E. Witten, Nucl. Phys. B **223**, 422 (1983) ; Nucl. Phys. B **223**, 433 (1983).
- [5] Ö. Kaymakçalan, S. Rajeev, and J. Schechter, Phys. Rev. D **30**, 594 (1984).
- [6] N.K. Pak and P. Rossi, Nucl. Phys. B *250*, 279 (1985).
- [7] S. Adler, Phys. Rev. **177**, 2426 (1969).
- [8] J.S. Bell and R. Jackiw, N. Cimento, **60A**, 47 (1969).
- [9] D.T. Son and M.A. Stephanov, Phys. Rev. D **77**, 014021 (2008).
- [10] R.L. Jaffe and A. Manohar, Nucl. Phys. B **337**, 509 (1990).
- [11] R. Jackiw, in *Current Algebra and Anomalies*, eds. S.B. Treiman et al., World Scientific, Singapore, 1985.
- [12] D.E. Kharzeev, H.-U. Yee, and I. Zahed, arXiv : 1104.0998 [hep-ph].